

TEXTURE SHADING: A NEW TECHNIQUE FOR DEPICTING TERRAIN RELIEF

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ABSTRACT

While conventional relief shading based on hillslope and illumination models has a long and successful history for visualization of mountainous topography, it also suffers certain limitations. Among these is anisotropy, or directional dependence, whereby the choice of lighting direction favors certain terrain features at the expense of others, according to their orientation. Another common drawback is a lack of visual hierarchy, since small terrain features affect slope and shading as much as large ones. Techniques such as adaptive illumination, relief generalization, resolution bumping, and atmospheric effect have been developed to mitigate these issues, while staying within the basic hillshading paradigm.

If we relax this constraint and consider a broader range of possibilities, and at the same time focus on isotropy and scale invariance as key design goals, new options for terrain presentation emerge. Texture shading provides one such solution, and is particularly useful for bringing out the drainage structure of terrain that is dominated by a strong ridge and canyon network. Texture shading produces a different view of the terrain than conventional hillshading; it can be used on its own to provide a novel shading technique, or it can be used to enhance traditional shaded relief by blending the two. Experience shows that many terrains benefit from a combination of a directional component, such as hillshading, and an isotropic component, such as texture shading.

Texture shading is produced by a mathematical process applied to a digital elevation model (DEM). The crux of the algorithm is a "fractional Laplacian" operator, which has been adapted to work on discrete data points instead of a continuous surface.

Keywords: shaded relief, fractional Laplacian, visual hierarchy, scale invariance, terrain maps.

INTRODUCTION

The goal of terrain presentation techniques such as shaded relief is to give a three-dimensional appearance to a map and to show information about the shape of the physical terrain. In the case of shaded relief (hillshading), this involves modeling the terrain as a surface illuminated by an imaginary light source, usually placed at the upper left of the image. The map then displays the shading, and sometimes shadows, that result from the interaction of the lighting on the terrain surface.

This process gives a terrain map that is very intuitive to understand, since it mimics the visual effect seen when viewing everyday objects. However, it also has drawbacks, which may make it difficult to perceive certain aspects of the terrain structure.

One such drawback that cartographers face is an inherent anisotropy, or directional dependence, of the shading effect, which is an unavoidable result of the illumination coming from a particular direction. For example, a ridge or canyon aligned perpendicular to the lighting direction has high contrast, since one side is well illuminated, facing the light, and the other side is darker, facing away. But a similar ridge or canyon aligned parallel to the light source is harder to discern on the map, because both sides are lighted similarly. This problem can be mitigated by algorithms that adaptively vary the illumination direction for different parts of the terrain; but anisotropic effects remain a challenge for mountain cartographers.

Another drawback of hillshading is a lack of visual hierarchy. Because shading is based on local slope, which depends only on the immediate neighborhood of each pixel, small terrain details can dominate the appearance of the map. Because of this, shaded relief is usually generalized by smoothing the terrain data and/or the shaded image to remove excess detail. This makes the larger terrain features more readable but eliminates the details altogether.

Patterson (2004) describes a technique called resolution bumping, which provides hillshading some visual hierarchy by blending two levels of detail but giving more weight to the smoothed version. This results in the smaller details being present but less pronounced. The present work aims, among other things, to expand this idea to more than two discrete levels of detail.

In this paper we examine an alternate approach to generating a grayscale-shaded image to represent terrain, rather than using illumination models.

ANALOGY TO STREET MAPS

Several useful features and cartographic techniques can be readily seen in a typical street map, such as the one shown in Figure 1.

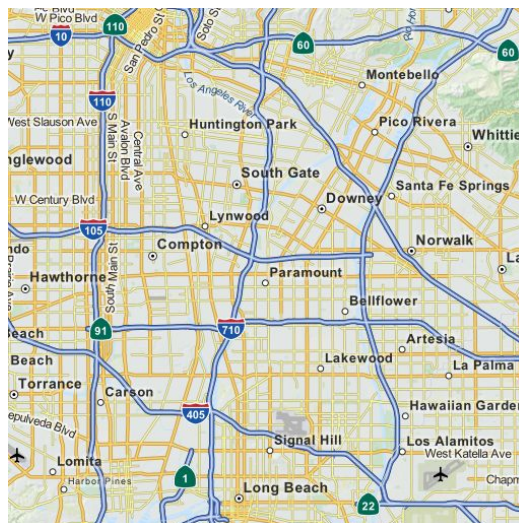


Figure 1. Street map example.

First, street maps by nature are built around a network of linear features (the roads), which then provides a skeleton for other map content – such as cities, parks, and other area and point features. This aids the map user in constructing a mental map of the city – the network forms a reference on which to mentally locate other features.

Likewise, the network structure shows connections among point features. Cities, for instance, are not isolated points on a map, but are connected by highways.

Second, street maps exhibit a clear visual hierarchy, which emphasizes the major components. In Figure 1, the most obvious features are the freeways, outlined and in bright blue, followed by secondary roads in more and more subdued colors. The key point here, which we will want to imitate with terrain maps, is that smaller details are shown with progressively less contrast.

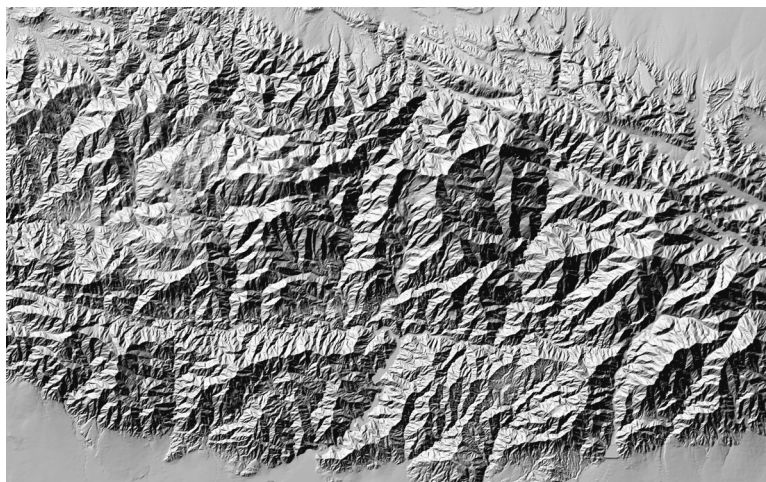
Terrain maps, on the other hand, are frequently lacking in these cartographic qualities. Consider the linear elements on a terrain map, such as a topographic map or hiking map, which typically include roads, trails, and stream lines. In mountainous areas roads tend to be sparse, and in the more rugged or remote areas, trails may be isolated as well. In addition, if the goal is to show the structure of the terrain, roads and trails may not show that well at all. Thus roads and trails are not good candidates for a network structure to represent the terrain.

Streams, however, do form a network (in fact, a hierarchical one) and do show important information about the terrain structure. But streams lines only show channels that contain water, thus missing canyons that are normally dry. Sources of elevation data often have a resolution smaller than the drainage density of the corresponding stream data, so at the largest map scales stream lines may not show all the terrain detail available. And the canyons and streams are only half the picture – between these are ridges and interfluves, which also form a network.

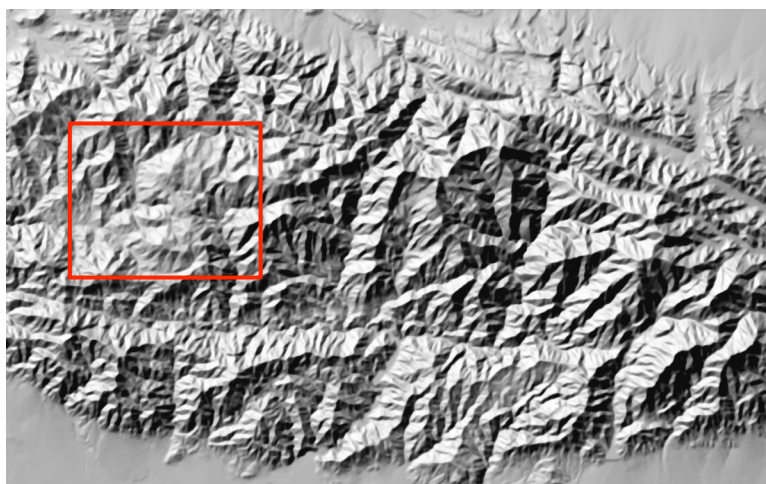
Some of the most commonly labeled natural features on a terrain map are mountain peaks. Unlike cities on a road map, peaks are usually shown as isolated, unrelated point features. In many cases, though, a peak is simply one of the high points on a ridge – or a point where ridges intersect. These peaks are, in fact, connected by the network of ridges, and so the ridge network becomes as important as the canyon network in understanding the structure of the landscape.

Figure 2 illustrates the lack of visual hierarchy in hillshaded relief maps in comparison to street maps. Figure 2(a) shows the cluttered appearance that can result from applying hillshading to the raw data; the details overwhelm. Figure 2(b) incorporates some relief generalization to reduce the clutter; but if we zoom in by enlarging the image, we see in Figure 2(c) that the relief has an unnaturally smooth appearance. At this scale it may be preferable to reduce the generalization to show details of a smaller size, as in Figure 2(d). Thus, the generalization is only appropriate for a single zoom level, and the generalized hillshading must be recomputed for each level of detail needed.

Ideally, we would like a way to make terrain maps that exhibit more of the desirable qualities we see in street maps.



(a) Hillshading without generalization



(b) Generalized hillshading



(c) Enlargement of image (b)



(d) Reduced generalization

Figure 2. Hillshading generalization.
San Gabriel Mountains, CA, USA.

DESIGN GOALS FOR NEW METHOD

To recap, some of the drawbacks of traditional hillshading include:

1. No obvious network structure,
2. No clear visual hierarchy, and
3. Orientation dependence.

These three problems then become three goals we want to achieve with a new terrain shading technique, which we will call “texture shading”:

1. Emphasizing the network structure formed by the canyons and ridges,
2. Scale invariance, which is a strong form of visual hierarchy, and
3. Orientation invariance (isotropy).

By scale invariance, we mean that the resulting image should have a consistent visual hierarchy across a wide range of scales. In particular, smaller and smaller details should have lower and lower contrast, where the ratio of contrast reduction should have a fixed relationship to the ratio of feature sizes. For example, if two similar features with size ratio 2:1 have a contrast ratio of 1.5:1, the same contrast ratio should hold regardless of whether the features are two large mountains or two small bumps on a mountain. This is akin to the self-similarity of fractals at different scales, and it fits well with the fractal-like qualities of real terrain.

One advantage of scale invariance is that it allows a single image to be useful at different scales. In the case of a large wall map, this means that the map maintains its visual hierarchy when viewed from across the room and also when viewed from up close, where the user is focused on only a small portion of the map. At a distance the smaller details, having less contrast, fade away, and the user sees only the major features, with the largest of those most prominent. As one walks toward the map, smaller features gradually come into view; and the smallest features visible always have the least contrast, maintaining the hierarchical character of the image.

In the case of a digital map, scale invariance means that the map can be zoomed to a larger scale without recalculating the shading – simply store a single high-resolution image, and reduce or enlarge the image according to the zoom level. (At higher zoom levels, the overall contrast of the image portion may be lower, since the local relief may be less pronounced, but the most significant features will still have the highest contrast. If desired, the overall contrast could simply be adjusted at each zoom level.) Likewise, the same visual hierarchy will be apparent regardless of the size of the displayed image – the same image could be viewed on a mobile phone, a desktop computer, or a large projection screen without degrading the effect.

These qualities are in contrast to traditional shading methods, where the appropriate amount of terrain generalization must be chosen according to the scale at which the map will be displayed; and changes in zoom level, image size, or expected viewing distance require corresponding changes in the level of detail to be included (see Figure 2).

Together, the three goals listed above form a very strong set of constraints, which considerably narrow the set of solutions. However, there are still a variety of options. For example, Kennelly and Stewart (2006) use diffuse illumination to produce an effect that meets these criteria whenever the sky model used is symmetric about the zenith – what they call a non-directional sky model (Kennelly and Stewart, 2014).

Since we have not required the new technique to be based on any sort of lighting model, we might expect other solutions whose results appear significantly different from traditional shaded relief techniques.

TEXTURE SHADING

One mathematical function that meets our design criteria and has the added advantage of being a linear operator (and thus computationally efficient), is called a fractional Laplacian.

The reader may be familiar with the Laplacian operator (named after the French mathematician Pierre-Simon Laplace), which is sometimes used in the field of image processing, for example as an edge detection technique. A fractional Laplacian, however, is a generalization of the ordinary Laplacian, is more complex to compute, and can exhibit much different behavior.

While the ordinary Laplacian is a second-derivative operator, the fractional Laplacian is a type of “fractional derivative.” It is, in fact, a family of different operators, since we can adjust the “fraction” to any desired value. For our purposes we’ll write this as a percentage, and we’ll assign 200% to the ordinary Laplacian, being a 2nd-order derivative. The values useful for terrain shading will mostly turn out to be in the range of about 50% to 100%. This parameter controls the level of detail – higher percentages give greater emphasis to terrain details, and relatively less emphasis to the major features and overall terrain structure.

The name “fractional” also suggests a connection to fractals, which accounts for the scale invariance property of the fractional Laplacian.

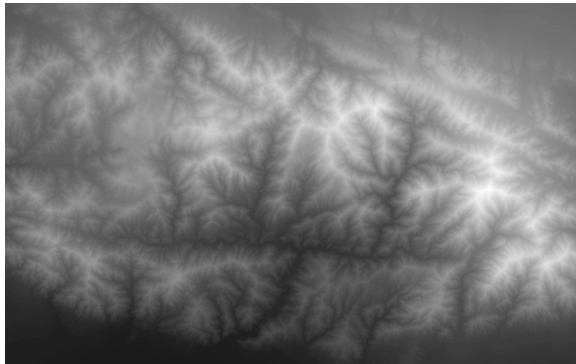
RESULTS

Figure 3 illustrates the results of the texture shading process. Figure 3(a) displays the terrain elevation directly as a grayscale – i.e., using hypsometric shading. This would be very similar to texture shading with a detail parameter of 0%. This rendering has somewhat the opposite problem as hillshading – the major drainage structure of the terrain is clear, but the details are completely lacking.

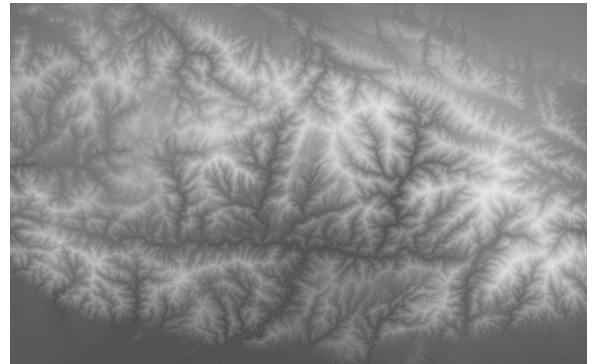
The remaining images in Figure 3 show the effect of increasing the percentage to 25%, 50%, and 75%. As the percentage increases, finer details become apparent. The cartographer can choose an appropriate value based on the intent of the map and the characteristics of the particular terrain.

Whereas Figure 3(a) displays absolute elevation (relative to sea level), the texture-shaded images show relative elevation, relative to the surrounding terrain. Light areas are higher than

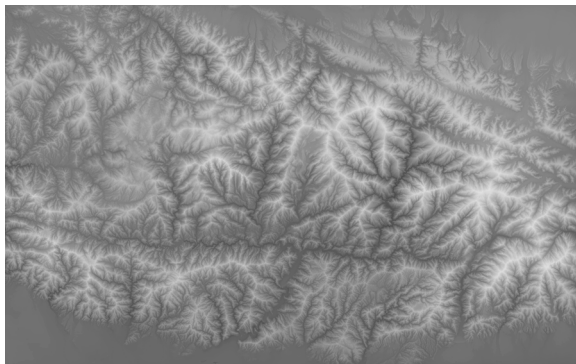
the surrounding terrain (i.e., ridges and peaks); dark areas are lower than the surrounding terrain (i.e., canyons and valleys).



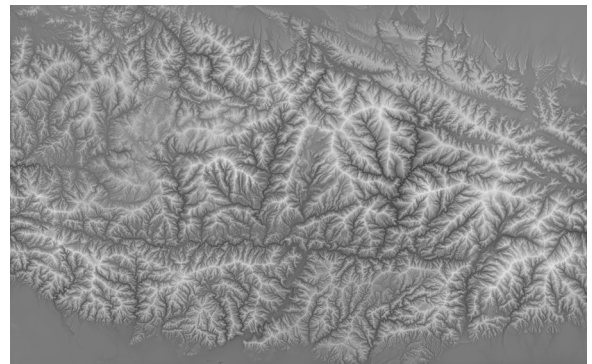
(a) DEM grayscale image



(b) Texture shading 25%



(c) Texture shading 50%

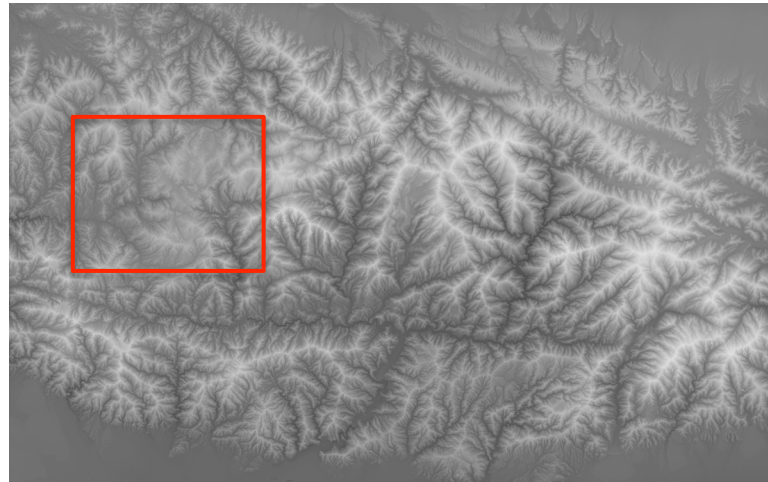


(d) Texture shading 75%

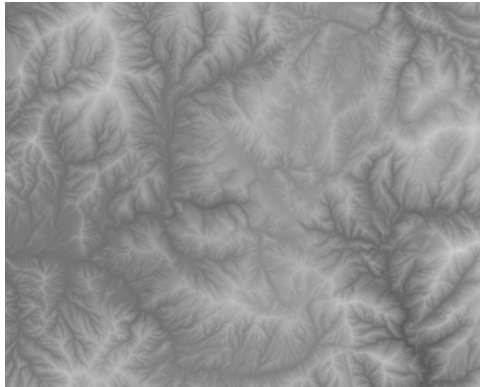
Figure 3. Texture shaded images with different values of the detail parameter.
San Gabriel Mountains, CA, USA.

Figure 4 illustrates the scale invariance property of the texture-shaded image. We enlarge the same region of the map as was done in Figure 2. However, unlike the hillshading example, the enlarged image still maintains a visual hierarchy, and still contains fine details. This is because even the smallest details are still present in the original image, except with reduced contrast so as not to clutter the image. But when those details are enlarged, they become visible. We might describe this effect as “self-generalization.”

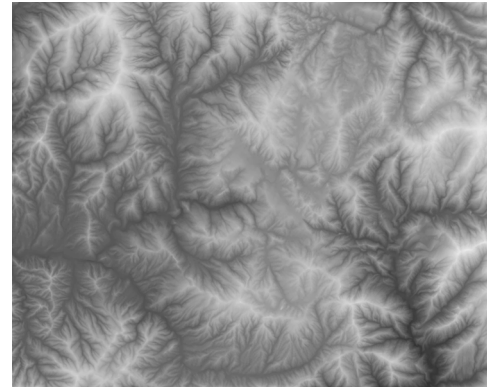
Since the overall vertical relief in the enlarged region of the terrain is less than in the mountain range as a whole, the enlarged image has a lower contrast. If desired, this can be adjusted by simply changing the contrast curve of the processed image, without having to recompute the texture shading. This is exactly analogous to applying a vertical exaggeration to the terrain – except that with texture shading this can be done after the shading is computed, whereas with hillshading it must be applied to the terrain data before doing the shading. (This difference is a consequence of the fractional Laplacian being a linear operator.)



(a) Texture shading 50%



(b) Enlargement of image (a)



(c) After adjusting contrast

Figure 4. Effect of scale change on texture shading.

Figure 5 focuses on a different area of the San Gabriel Mountains, where there are several named peaks and saddle points. As road maps show cities being connected by highways, notice how texture shading makes clear the network of ridges connecting the peaks and saddles, making it easier to construct a mental framework for the terrain structure. Thus we have achieved some success in bringing this characteristic of street maps into terrain mapping.

Figure 6 shows two maps with texture shading that also incorporate hypsometric tinting. Figure 6(a) was produced using a detail setting of 100%. The higher value in this case is needed because the cliffs along the valley walls are so much taller than most of the other terrain features; at lower percentages the cliffs dominate the image, leaving very little contrast in the rest of the map. The higher percentage attenuates the valley walls in relation to the smaller features. As a result the banks of the river meandering the valley, which are only a few meters high, are visible in the image, as well as the 1000-meter cliffs along the valley walls and the face of Half Dome at the right. Texture shading ensures there is a brightness differential across any sudden change in elevation – the top of a cliff is highlighted (being higher than the neighboring terrain), and the bottom of a cliff is darkened, as if in shadow.

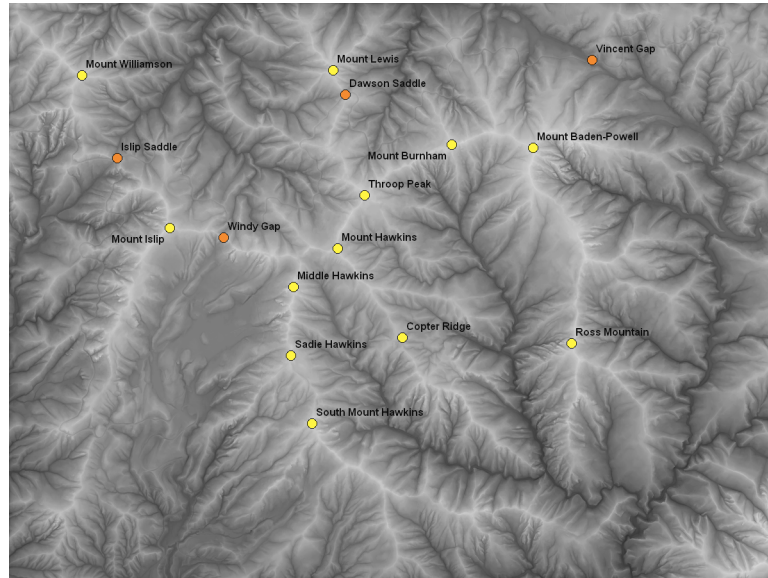
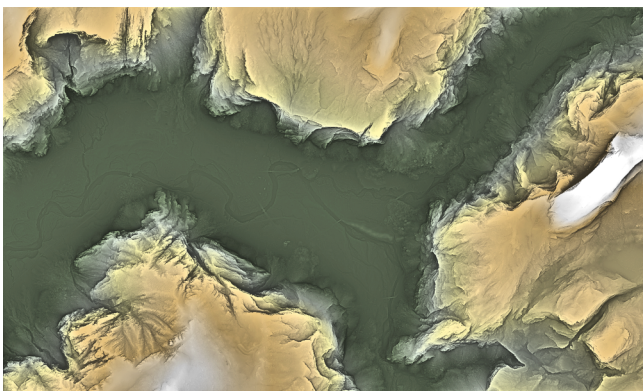
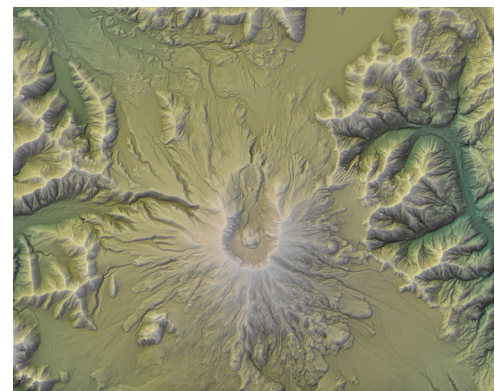


Figure 5. Section of San Gabriel Mountains showing named peaks and saddles.

Texture shading seems to perform well on a variety of terrain types, and is especially useful for bringing out the drainage network of the terrain. However, in some cases it fails to produce a three-dimensional appearance to the image – most notably, with volcanic peaks. Kennelly and Stewart (2014) indicate a similar experience with their non-directional sky models. This suggests the criterion of isotropic shading may not always be the right goal. Instead, perhaps the best shading results require a combination of isotropic and anisotropic (non-directional and directional) shading components, and results suffer when either one is omitted. In fact, by using diffuse lighting, even Kennelly and Stewart’s directional models incorporate a non-directional component, unlike traditional hillshading which uses a single point lighting source.



(a) Eastern part of Yosemite Valley, CA, USA



(b) Mount St. Helens, WA, USA

Figure 6. Texture shaded images combined with hypsometric tints.
Image (b) is also blended with traditional hillshading.

An easy way to add the missing directional component to texture shading is to blend it with traditional shaded relief. This idea was first suggested by Patterson (2013). It has the added advantage of making the appearance of the map more similar to what map readers are used to, which may help the untrained user to interpret the terrain. The final example in Figure 6(b) is

blended with a traditional hillshaded image. The result seems to retain the positive aspects of both components – the texture shading accentuates the drainage structure, while the hillshading ensures a conical appearance for the volcano. Thus, while texture shading can be used as a standalone shading technique, it is often beneficial to use it in combination with more traditional relief shading.

Software for performing the texture shading algorithm, as well as further information and examples, is available at <http://www.textureshading.com/>.

IMPLEMENTATION NOTES

Certain mathematical difficulties arise in making a practical implementation of a fractional Laplacian algorithm for terrain data. We'll give a summary here of how those challenges have been overcome, for readers who are interested in such details.

First is the problem of how to discretize the fractional Laplacian operator. This issue stems from the fact that the operator is defined on a function of continuous variables, but the elevation data is known only at discrete points on the terrain. A similar issue exists in many other cases of terrain or image processing – for example, a Gaussian blur operator. However, the fractional Laplacian has both a singularity and a slow decay rate that make the situation considerably more difficult.

In the spatial domain, the fractional Laplacian of a function $f(x, y)$ can be written as

$$[(-\Delta)^{d/2}f](x, y) = k_d \iint_{\mathbb{R}^2} \frac{f(x, y) - f(x + u, y + v)}{(u^2 + v^2)^{d/2+1}} du dv \quad (1)$$

where d is the fractional order (the percentage), and k_d is a constant depending on d .

Notice that the integral describes a weighted sum of the elevation differences between a particular point and its surrounding terrain, giving more weight to the nearby terrain. The power law in the denominator is what accounts for the scale invariance property, and the exponent determines the relative importance given to nearby differences (details).

The most direct approach to discretize the operator would be to approximate the integral by evaluating the integrand at each pixel of the terrain data and summing. However, at the point $u = v = 0$, both the numerator and denominator of the integrand are zero. The limit of the integrand at this point can be infinite even though the integral is a well-defined finite value. And if we simply ignore that point, then we may be leaving out the largest term in the sum.

We use a different approach. Instead of modifying the operator to act on discrete data, we modify the discrete data to be continuous, and then apply the continuous operator. In other words, we construct, at least abstractly, a continuous terrain surface from the discrete elevation points – that is, we interpolate the data. A smooth interpolator is needed; sharp corners in the interpolated surface will result in divergent integrals (infinite results). An example of an interpolator that meets our needs is a bicubic spline. Experience has shown that the final

shading results are not highly sensitive to the exact choice of interpolator, except at very high fractional orders (close to 200%).

Both the interpolation and the fractional Laplacian will be more easily performed – and far more efficiently – in the spatial frequency domain, by means of a Fourier transform. It is a little-known fact that a cubic spline on regularly spaced data can be written as a convolution (Dodgson, 1992, pp. 127-130), and thus computed quickly in the frequency domain by means of a multiplier function (at least if we ignore edge effects, or assume the data has infinite spatial extent, which we will do for the moment).

After taking the Fourier transform of the data, a cubic spline interpolation consists of multiplying the transformed data by this function:

$$s(v) = \frac{3 \sin^4 \pi v}{\pi^4 (2 + \cos 2\pi v)} \cdot v^{-4} \quad (2)$$

where v is the frequency in units of cycles per pixel. For a bicubic spline we multiply our two-dimensional data by

$$R(v_x, v_y) = s(v_x) \cdot s(v_y) \quad (3)$$

using the x and y components of the spatial frequency. Then the fractional Laplacian is just a multiplication by

$$T(v_x, v_y) = (2\pi)^d \left[\left(\frac{v_x}{\Delta x} \right)^2 + \left(\frac{v_y}{\Delta y} \right)^2 \right]^{d/2} \quad (4)$$

where Δx and Δy are the pixel dimensions in distance units (e.g., meters). Here we can see that the fractional Laplacian can also be described as a scale-independent high-pass or sharpening filter, comparatively enhancing higher spatial frequencies (narrow terrain features) and attenuating lower frequencies (wide features).

The Fourier transform of the original discrete data was periodic; by multiplying by non-periodic functions, we now have a non-periodic Fourier transform that represents a continuous function in the spatial domain. Before transforming back, we need our function to represent a discretely sampled dataset again. The frequency-domain analog of spatial sampling is to sum the function over all integer offsets, like this:

$$F(v_x, v_y) \mapsto \sum_{i \in \mathbb{Z}} \sum_{j \in \mathbb{Z}} F(v_x + i, v_y + j) \quad (5)$$

Unfortunately this leaves us with an infinite sum that probably has no closed-form solution. But notice that the cubic spline spectrum $s(v)$ decays very rapidly; it's nearly zero for $|v| > 1$ (see Figure 7).

We could simply ignore those small values, but that amounts to replacing the cubic spline with an “interpolator” that doesn’t quite pass through the spatial data points. A better option would be to use a function that closely approximates $s(v)$ but still provides a true interpolator, is identically zero for $|v| > 1$ (i.e., bandlimited to the sampling frequency), splices smoothly at $|v| = 1$ (which minimizes edge artifacts in the spatial domain), and is no more difficult to compute. Trial and error has found a simple function that meets these needs:

$$s_2(v) = \frac{(1 - |v|)^4}{(1 - |v|)^4 + |v|^4}, |v| \leq 1 \quad (6)$$

By replacing $s(v)$ with $s_2(v)$, the number of nonzero terms in the double sum of equation (5) becomes finite (only four terms remain), so the entire process becomes feasible. The final step in the discretized fractional Laplacian is to perform another Fourier transform back to the spatial domain.

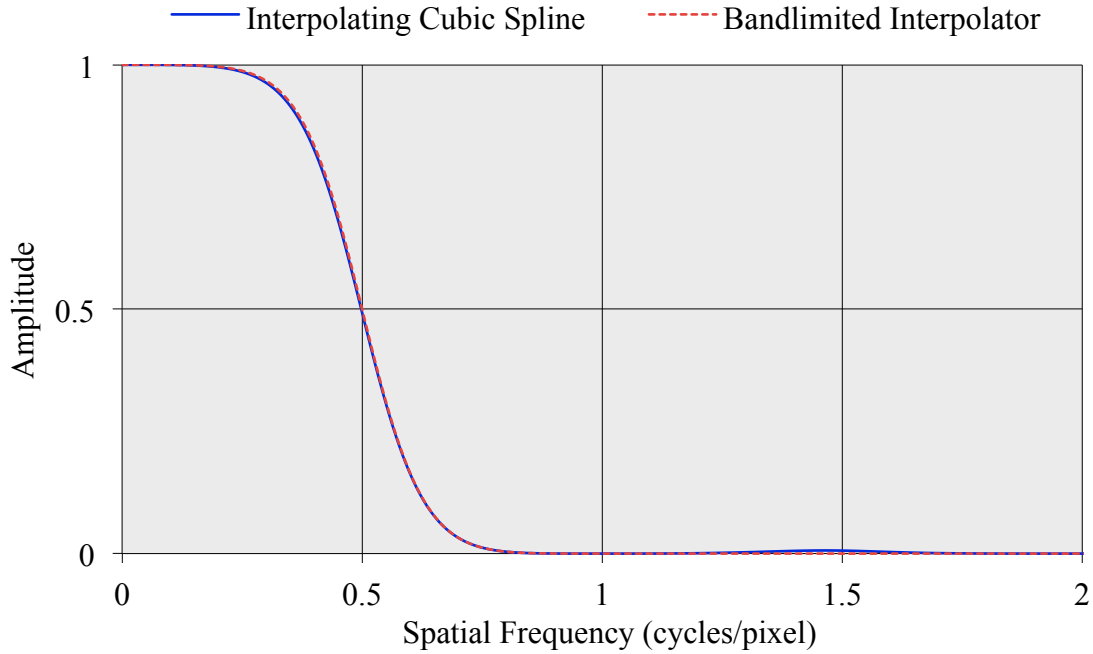


Figure 7. Comparison of interpolator spectra. The cubic spline’s frequency-domain representation, $s(v)$, has small high-frequency components, as can be seen near $v = 1.5$. The function $s_2(v)$ is bandlimited at $v = 1$ and very nearly matches the cubic spline.

Up until now we have assumed the data extends infinitely in all directions, so as to ignore edge effects. Returning to equation (1), we see unfortunately that the rate of decay of the contribution from distant pixels is slow, so that the results will apparently depend on data beyond the edges of the map. While it is true that the numerical values will depend on the missing data, the visual character is most affected by closer pixels. Because of this we can safely ignore this problem as long as we can artificially extend the data beyond the edges of the map in a way that resembles real terrain – i.e., it avoids discontinuities and does not

diverge in elevation at large distances. This is easily accomplished by reflecting the terrain across the map boundaries, a technique often used in image processing. By using a discrete cosine transform (DCT) as the Fourier transform algorithm, this reflected boundary condition is implicitly incorporated with no additional work necessary.

It should be noted that, although this process is generally sufficient to avoid visual artifacts on an individual map, adjacent map tiles that are processed separately will not match seamlessly. This is because each tile is effectively generating different artificial data for the surrounding areas, and that data is affecting the respective results. To generate seamless map tiles, the entire region should be processed as one large map and then the single image divided into tiles.

Finally, after the fractional Laplacian is computed, the value at each pixel must be converted to a grayscale value ranging from black to white. This is known as a contrast stretch. There are two problems here. First, the output values are neither dimensionless values (like percentages), nor distance measurements (like elevation), and there is no limit on the range of possible values. Each terrain generates a unique distribution of values, and it is very difficult to automatically select a contrast setting; for best results, this must be adjusted manually. Second, there tend to be a large number of outlier values, and some of these outliers are important to the look of the map. Simply clipping the outliers at black for low values and white for high values does not generally give a satisfactory appearance. Thus, a simple linear contrast stretch with the outliers clipped is not ideal; more pleasing results come from using a smooth “S” curve that saturates to black or white gradually instead of abruptly. This allows the outliers to retain some contrast.

Two similar S curves that work well for conversion to grayscale ($-1 = \text{black}$, $+1 = \text{white}$) are

$$z \mapsto \tanh(az + b) \quad (7)$$

and

$$z \mapsto \frac{(az + b)}{\sqrt{1 + (az + b)^2}} \quad (8)$$

where a is a contrast parameter that trades off contrast in the outlier pixels (which are the ridges and canyons) against contrast in the midtones, and b is an optional midtone brightness parameter. Adjustments to the contrast parameter can also be used to serve a purpose similar to the vertical enhancement that is sometimes applied to elevation data when generating traditional shaded relief.

ACKNOWLEDGEMENTS

Street map © OpenStreetMap contributors, <http://www.openstreetmap.org/copyright>.
 Tiles courtesy of MapQuest, <http://www.mapquest.com/>.

Yosemite Valley elevation data is courtesy of the U.S. National Park Service.

Elevation data for Figure 5 is courtesy of the Los Angeles Region Imagery Acquisition Consortium (LAR-IAC) and Infotech Enterprises America, Inc., via the U.S. Geological Survey.

All other elevation data used herein is courtesy of the U.S. National Elevation Dataset, <http://ned.usgs.gov/>.

The author is grateful to Tom Patterson of the U.S. National Park Service and Brett Casebolt of Natural Graphics for their encouragement and suggestions, and especially their practical help in developing the grayscale conversion function; and to James Meacham of the University of Oregon for his suggestion that inspired the midtone brightness adjustment feature.

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